OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4721**: Core Mathematics 1

Advanced Subsidiary GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

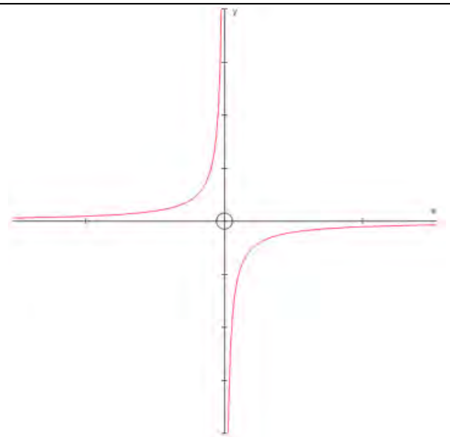
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

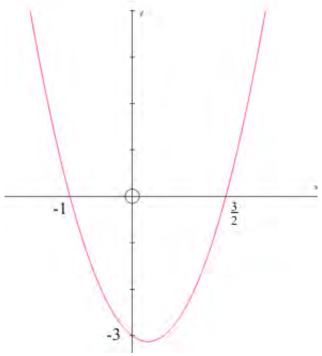
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\frac{8}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{8\sqrt{3}+8}{3-1}$ $4\sqrt{3}+4$	M1 A1 A1 **[3]**	Multiply top and bottom by $\sqrt{3}+1$ or $-\sqrt{3}-1$ – evidence of multiplying out needed Either numerator or denominator correct Final answer **cao**	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{3}+b$ **A1** Either a or b correct **A1** Both correct
2	(i)			B2 **[2]**	Excellent curve in both quadrants: • correct shape, symmetrical, not touching axes • asymptotes clearly the axes • not finite • allow slight movement away from asymptote at one end but not more.	**N.B. Ignore 'feathering' now that answers are scanned.** **B1** only – correct shape in 2nd and 4th quadrants **only**. Graph must not touch axes more than once. Finite "plotting" condoned.
	(ii)		$y=-\frac{1}{x-2}$ oe	M1 A1 **[2]**	$(y=)-\frac{1}{x-2}$ or $(y=)-\frac{1}{x+2}$ Fully correct, must include "$y=$"	$(y=)\frac{1}{x+2}$ or $(y=)\frac{1}{x-2}$ is **M0**
	(iii)		Stretch Scale factor $\frac{1}{3}$ parallel to the x-axis (or y-axis)	B1 B1 **[2]**	Stretch or "stretched" etc.; **do not accept** squashed, compressed, enlarged etc. Correct description Condone just "factor $\frac{1}{3}$" but **no reference to units.** Must not follow e.g. "reflection"	**0/2** if more than one type of transformation mentioned **ISW** non-contradictory statements For "parallel to the x/y axis" allow "vertically", "in the x/y direction". **Do not accept** "in/on/ across/up/along/to/towards the $x/\ y$ axis"

Question			Answer	Marks	Guidance	
3	(i)		5^8	B1 [1]	**cao**	
	(ii)		$5^{-\frac{1}{4}}$	M1 A1 [2]	Fourth root ≡ $\frac{1}{4}$ soi **cao www**	
	(iii)		$5^{\frac{9}{2}}$	M1 A1 [2]	$(5^{\frac{3}{2}})^3$ or $5^3 \times 5^{\frac{3}{2}}$ or other correct product of two simplified powers of 5 oe **cao www**	
4			$k = x^{\frac{1}{3}}$	M1*	Use a substitution to obtain a quadratic, or factorise into 2 brackets each containing $x^{\frac{1}{3}}$	**No marks** if whole equation cubed/ rooted etc. **No marks** if straight to quadratic formula with no evidence of substitution at start and no cube rooting/cubing at end.
			$k^2 - k - 6 = 0$ $(k-3)(k+2) = 0$	M1dep	Attempt to solve resulting three-term quadratic – **see guidance in appendix 1**	
			$k = 3, k = -2$	A1	Correct values of k	**Spotted solutions:** If **M0 DMO** or **M1 DM0** **SC B1** $x = 27$ **www** **SC B1** $x = -8$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)
			$x = 3^3, x = -2^3$	M1	Attempt to cube at least one value	
			$x = 27, x = -8$	A1 [5]	Final answers correct **ISW**	
5	(i)		$AB = \sqrt{(5-2)^2 + (-3-1)^2}$	M1	Attempt to use Pythagoras' theorem – 3/4 numbers substituted correctly **and attempt to square root**	
			$AB = 5$	A1 [2]	Final answer correct, must be fully processed. ±5 is A0.	

Question			Answer	Marks	Guidance	
	(ii)		$\left(\frac{2+5}{2}, \frac{1+-3}{2}\right)$	M1	Correct method to find mid-point of line	Alternative using general point on the perpendicular
			(3.5, –1)	A1		**M2** States P (x, y) a point on the perpendicular and attempts PA = PB or $PA^2 = PB^2$
			Gradient of AB = $-\frac{4}{3}$	B1	Processed	**A1** At least one of PA, PB correct
			Perpendicular gradient = $\frac{3}{4}$	B1**ft**	$\frac{-1}{\text{their gradient}}$ processed	**A1** Both correct **M1** Expands and simplifies
			$y+1=\frac{3}{4}(x-\frac{7}{2})$	M1 A1	Equation of straight line through their mid-point, any non-zero gradient in any form	**A1** Correct equation found **A1** Correct equation in required form
			$6x-8y-29=0$	A1 **[7]**	**cao** Must be correct equation in required form i.e. $k(6x-8y-29)=0$ for integer k. **Must have "=0"**	
6			$x^2 - (5-2x)^2 = 3$	M1*	Substitute for x/y or valid attempt to eliminate one of the variables	If y eliminated: $3y^2 + 10y - 13 = 0$
			$3x^2 - 20x + 28 = 0$	A1	Three term quadratic in solvable form	$(3y + 13)(x - 1) = 0$
			$(3x - 14)(x - 2) = 0$	M1dep	Correct method to solve three term quadratic – **see appendix 1**	**Spotted solutions: If M*0** **SC B1** $x = 2, y = 1$ **www**
			$x=\frac{14}{3}, x=2$	A1	Both x values correct	**SC B1** $x=\frac{14}{3}, y=-\frac{13}{3}$ **www** Must show on both line and curve
			$y=-\frac{13}{3}, y=1$	A1 **[5]**	Both y values correct. **Allow 1 A mark for one correct pair of x and y from correct factorisation.**	(Can then get 5/5 if both found **www** and exactly two solutions justified)
7	(a)		$(x^2 + 3)(5 - x) = 5x^2 - x^3 + 15 - 3x$	M1	Attempt to multiply out brackets, Must have four terms, at least three correct	<u>Alternative using product rule:</u> Clear attempt at correct rule **M1*** Both expressions fully correct **A1**
				A1	Fully correct expression. Do not ISW if signs then changed. Max 2/4.	Expand brackets of both parts **M1*dep**
			$\frac{dy}{dx} = 10x - 3x^2 - 3$	M1 A1 **[4]**	Attempt to differentiate their expression, (power of at least one term involving x reduced by one)	Fully correct expression **A1**

Question		Answer	Marks	Guidance	
	(b)	$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{4}{3}}$	M1 A1	Attempt to differentiate i.e. $-\frac{1}{3}x^{-\frac{k}{3}}$ soi for positive integer k Fully correct	$x^{-\frac{1}{3}}$ misread as $x^{\frac{1}{3}}$ earns max 2/4: $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ **M1 A0 MR**
		When $x = -8$ $\frac{dy}{dx} = -\frac{1}{3} \times (-8)^{-\frac{4}{3}}$	B1	$(-8)^{-\frac{4}{3}} = \frac{1}{16}$ **www** Must use – 8	$(-8)^{-\frac{2}{3}} = \frac{1}{4}$ **B1**
		$\frac{dy}{dx} = -\frac{1}{3} \times \frac{1}{16} = -\frac{1}{48}$	A1 **[4]**	Final answer	Final answer $\frac{1}{12}$ **A0 MR**
8	(i)	$(2x-3)(x+1) = 0$	M1	Correct method to find roots – **see appendix 1**	
		$x = \frac{3}{2}, x = -1$	A1	Correct roots	
			A1**ft** B1 **[4]**	Good curve: • Correct shape, symmetrical positive quadratic • Minimum point in the correct quadrant for their roots **(ft)** • their x intercepts correctly labelled **(ft)** y intercept at $(0, -3)$. Must have a graph.	
8	(ii)	$x < -1, x > \frac{3}{2}$	M1 A1**ft** **[2]**	Chooses the "outside region" Follow through x-values in (i). Allow "$x < -1, x > \frac{3}{2}$", "$x < -1$ **or** $x > \frac{3}{2}$" but do not allow "$x < -1$ **and** $x > \frac{3}{2}$"	If restarted, fully correct method for solving a quadratic inequality including choosing "outside region" needed for **M1** **NB** e.g. $-1 > x > \frac{3}{2}$ scores **M1A0** Must be strict inequalities for **A** mark

Question			Answer	Marks	Guidance	
8	(iii)		$b^2 - 4ac = 1^2 - 4 \times 2 \times -(3 + k)$	M1	Rearrangement and use of $b^2 - 4ac < 0$, must involve 3 and k in constant term (not $3k$)	Alt for first two marks: **M1** Attempt to find turning point and form inequality $k < y_{min}$
			$25 + 8k < 0$	A1	$p + 8k < 0$ oe found, any constant p. p need not be simplified	**A1** turning point correct $(\frac{1}{4}, -\frac{25}{8})$
			$k < -\frac{25}{8}$	A1 **[3]**	Correct final answer	**If M0** (either scheme) **SC B1** $k = -\frac{25}{8}$ or $k > -\frac{25}{8}$ seen
9	(i)		$\frac{dy}{dx} = 6x^2 - 2ax + 8$	M1 A1	Attempt to differentiate, at least two non-zero terms correct Fully correct	
			When $x = 4$, $\frac{dy}{dx} = 104 - 8a$	M1	Substitutes $x = 4$ into their $\frac{dy}{dx}$	These Ms may be awarded in either order
			$\frac{dy}{dx} = 0$ gives $a = 13$	M1 A1 **[5]**	Sets their $\frac{dy}{dx}$ to 0. Must be seen	
	(ii)		$\frac{d^2y}{dx^2} = 12x - 26$	M1	Correct method to find nature of stationary point e.g. substituting $x = 4$ into second derivative (at least one term correct from their first derivative in (i)) and consider the sign	**Alternate valid methods include:** 1) Evaluating gradient at either side of 4($x > \frac{1}{3}$) e.g. at 3, –16 at 5, 28 2) Evaluating $y = -46$ at 4 and either side of 4 ($x > \frac{1}{3}$) e.g. (3, – 37), (5, – 33)
			When $x = 4$, $\frac{d^2y}{dx^2} > 0$ so minimum	A1 **[2]**	**www**	If using alternatives, working must be fully correct to obtain the **A** mark
	(iii)		$6x^2 - 26x + 8 = 0$ $(3x - 1)(x - 4) = 0$ $x = \frac{1}{3}$	M1 M1 A1 **[3]**	Sets their derivative to zero Correct method to solve quadratic **(appx 1)** oe	 Could be $(6x - 2)(x - 4) = 0$ or $(3x - 1)(2x - 8) = 0$

Question			Answer	Marks	Guidance	
10	(i)		$C = (5, -2)$	B1	Correct centre	
			$(x-5)^2 + (y+2)^2 - 25 = 0$	M1	$(x \pm 5)^2 - 5^2$ and $(y \pm 2)^2 - 2^2$ seen (or implied by correct answer)	Or attempt at $r^2 = f^2 + g^2 - c$
			Radius = 5	A1 **[3]**	Correct radius – do not allow **A** mark from $(x+5)^2$ and/or $(y-2)^2$	± 5 or $\sqrt{25}$ A0.
10	(ii)		Gradient $PC = \dfrac{2--2}{8-5} = \dfrac{4}{3}$	M1 A1	Attempt to find gradient of radius (3/4 correct)	**See also alternative methods on next page**
			Gradient of tangent $= -\dfrac{3}{4}$	B1**ft**	$\dfrac{-1}{\text{their gradient}}$ processed	
			$y - 2 = -\dfrac{3}{4}(x-8)$	M1	Equation of straight line through P, using their perpendicular gradient (not from rearrangement)	Do not allow use of gradient of radius instead of tangent
			$4y + 3x = 32$	A1 **[5]**	Rearrange to required form **www AG**	Ignore order of terms
PLEASE SEE NEXT PAGE FOR 10ii ALTERNATIVE METHODS						
	(iii)		$Q = (0, -2)$	B1	Q found correctly	For the M mark, allow splitting into two triangles $\dfrac{1}{2} \times 6 \times 8 + \dfrac{1}{2} \times 4 \times 8$
			$R = (0, 8)$	B1	R found correctly	
			Area $= \dfrac{1}{2} \times (8--2) \times 8$	M1	Attempt to find area of triangle with their Q, R and height 8 i.e. $\dfrac{1}{2} \times (y_R - y_Q) \times 8$	If using PQ as base then expect to see $\dfrac{1}{2} \times \sqrt{80} \times \sqrt{80}$ **www**
			40	A1 **[4]**		

<table>
<tr><th colspan="3">Alternative methods for 10(ii)</th></tr>
<tr><th>Alternative by rearrangement</th><th>Alternative for equating given line to circle</th><th>Alternative for implicit differentiation:</th></tr>
<tr>
<td>Gradient of radius = $\frac{2--2}{8-5}=\frac{4}{3}$ M1A1

Attempts to rearrange equation of line to find gradient of line = $-\frac{3}{4}$ and compares with gradient of radius M1

Multiply gradients to get –1 B1

Check (8, 2) lies on line B1</td>
<td>Substitute for x/y or attempt to get an equation in 1 variable only M1

$k(x^2 - 16x + 64) = 0$ or $k(y^2 - 4y + 4) = 0$ A1

Correct method to solve quadratic – see appendix 1 M1

$x = 8$, $y = 2$ found A1

States one root implies tangent B1</td>
<td>M*1 Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term
A1 $2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$
A1 Substitution of (8, 2) to obtain $-\frac{3}{4}$
Then as main scheme OR
Attempts to rearrange equation of line to find gradient of line = $-\frac{3}{4}$ M1dep
Check (8, 2) lies on line B1</td>
</tr>
</table>

APPENDIX 1

Solving a quadratic

This is particularly important to mark correctly as it features several times on the paper.
Consider the equation: $3x^2 - 13x - 10 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(3x + 5)(x - 2)$	**M1**	$3x^2$ and -10 obtained from expansion
$(3x - 4)(x - 3)$	**M1**	$3x^2$ and $-13x$ obtained from expansion
$(3x + 5)(x + 2)$	**M0**	only $3x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for *a* or *b* or *c* scores **M0**

$$\frac{-13 \pm \sqrt{(-13)^2 - 4\times 3\times -10}}{2\times 3}$$ earns **M1** (minus sign incorrect at start of formula)

$$\frac{13 \pm \sqrt{(-13)^2 - 4\times 3\times 10}}{2\times 3}$$ earns **M1** (10 for *c* instead of −10 is the only sign slip)

$$\frac{-13 \pm \sqrt{(-13)^2 - 4\times 3\times 10}}{2\times 3}$$ **M0** (2 sign errors: initial sign and *c* incorrect)

$$\frac{13 \pm \sqrt{(-13)^2 - 4\times 3\times -10}}{2\times -10}$$ **M0** ($2c$ on the denominator instead of $2a$)

Notes – for equations such as $3x^2 - 13x - 10 = 0$, then $b^2 = 13^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the “square root stage” involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$3x^2 - 13x - 10 = 0$$

$$3\left(x^2 - \frac{13}{3}x\right) - 10 = 0$$

$$3\left[\left(x - \frac{13}{6}\right)^2 - \frac{169}{36}\right] - 10 = 0$$

$$\left(x - \frac{13}{6}\right)^2 = \frac{289}{36}$$

$$x - \frac{13}{6} = \pm\sqrt{\frac{289}{36}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{13}{6}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.

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